## 8.1 Probability Models and Rules

Probability: The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

- A repeatable phenomenon is **random** if any particular outcome is quite unpredictable, while in the long, run after a large number of repeated trials, a regular, predictable pattern emerges

**Probability Model**: is a mathematical description of a random phenomenon consisting of two parts as follows:

- a **sample space, S**
  The **sample space**, S, of a random phenomenon is the set of all possible outcomes. An **event** is any outcome (element of sample space) or set of outcomes (subset of sample space) of a random phenomenon

  Example of Sample Space:
  - When we toss a coin once, there are only two outcomes, heads and tails. So the sample space is S = {H, T}.
  - If we draw a random sample of 1000 U.S. residents age 18 and over, as opinion polls often do, the sample space contains all possible choices of 1000 of the more than 230 million adults in the country.

- a way of assigning **probabilities** to events
  The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions. If *A* and *B* are events in sample space S, and P(A) is the probability of that event *A* will occur and P(B) is the probability that event *B* will occur, then the following rules hold true.

  **Probability Rules**:
  1. **$0 \le P(A) \le 1$: Any probability is a number between 0 and 1, inclusively.** An event with probability 0 never occurs,

and an event with probability 1 always occurs, and an event with probability 0.5 occurs half the trials in the long run.

2. **P(S) = 1: All possible outcomes together must have probability 1.** Because some outcomes must occur on every trial, the sum of the probabilities for all possible (simpliest) outcomes must be exactly 1.
3. **$P(A^c) = 1 - P(A)$: The probability that an event does not occur is 1 minus the probability that the event does occur. This is known as the complement rule.** If an event occurs in (say) 70% of the trials, then it fails to occur in the other 30%. The probability that an event occurs and the probability that it does not occur always adds up to 100%, or 1.
4. **If two events (A and B) are independent of one another, then the probability that one and the other occurs is the product of their individual probabilities. I.E. P(A and B) = P(A) x P(B). This is the multiplication rule for independent events.**

Independent event Example:
Pair-a-Dice: Outcomes for Rolling Two Dice. What is the sample space for rolling two dice?

- The probability for each outcome on each dice {1, 2, 3, 4, 5, 6}= 1/36
- Since they are independent events (outcome of one event doesn't affect outcome of the other event) use **Rule #4**. P(A and B) = P(A) x P(B) = (1/6) X (1/6) = (1/36)

5. **P( A or B) = P(A) + P(B) – P (A ad B): The probability that an event or another event occurs is the sum of the individual probabilities minus the probability of the overlap, This is the general addition rule.**
6. **If two events (A and B) have no outcomes in common (disjoint), then the probability that one or the other occurs is the sum of their individual probabilities. I.E. P(A or B) = P(A) + P(B). This is the addition rule for disjoint events. If the two events cannot occur simultaneously, then P(A and B) = 0.**

Tossing Two Coins: In two coin tosses, what is the probability that heads will appear at least once?

- Sample space: {HH, HT, TH, TT}
  - Probability that heads will appear at least once: (3/4)

**Complement of an Event**: Complement of *A* is the event that *A* does *not* occur, written as $A^c$.

**Disjoint Events**: Two events are **disjoint events** if they have no outcomes in common. Disjoint events are also called ***mutually exclusive events***.

**Independent Events**: Two events are **independent events** if the occurrence of one events has no effect on the probability of the occurrence of the other event.