Probability: is the number of ways that a certain outcome in a random experiment can occur divided by the total number of possible outcomes.

Permutation: a rearrangement of a certain number of items that are chosen without replacement

- Formula for counting the number of permutations of k items chose from n items (without replacement is nPk = n!/((n-k)!), where **n** is the total number of items in the population/sample and **k** is the number of items being selected from the population/sample.
  - Number of ways to arrange all the n items and divide by the number of ways to rearrange the items you didn't select. Because you select k items, you leave (n-k) items unselected.
  - If you're rearranging all n items chosen from n items. This equals nPn, which just becomes n!.

Suppose you have six people in the group going into a movie theater, but you can select only four of them to sit in a row together. How many ways to do you have to select the four people and rearrange them in one row of the theater?

- (ANSWER: 360 ways to choose 4 people from the group of 6 to rearrange those 4 people in a row together)

Suppose the people you choose to go to the movie are Tim, Syd, Elena, and Mark. How many different ways can they sit together in one row (straight line) in the movie theater?

- (ANSWER: 24 ways)

Wedding Planner:
Dilemma: There are always people who require some special consideration in the effort to keep peace.

Suppose you have four friends named Jim, Arun, Soma, and Eric. How many ways can you rearrange the individuals in a row so that Soma and Eric don't sit next to each other?

- You have 4! = 24 possible ways to rearrange all four of them.
- 6 scenarios involve Eric and Soma sitting next to each other.
- Therefore 24-8=18 scenarios involve the two not sitting next to each other

How many ways can you rearrange the letters of the word "Mississippi?" The word contains 11 letter, but not all the letters are distinct—you see four Ss, two Ps, and four Is to go along with the M.

- You have 11! ways to arrange all 11 items, you have to account for arrangements where like letters are interchanged and the word is exactly the same; for example, changing the two Ps around doesn't do anything.
- You take the 11! ways and dividing by the number of rearrangements of the like letters doe so for each letter. In the case of Mississippi, the answer is 11!/(4!2!4!),

because the word has 4 Ss, two Ps, and four Is who individual rearrangement don't really count.
- **34,650 rearrangements**

Suppose you have four friends—Tim, Syd Elena, and Mark—who go to the ice cream shop after a movie, and they sit at a round table. How many ways can they arrange themselves in a circle? (differs because a circle does not have a clear beginning and end)

- In general you have (k-1)! possible ways to rearrange k items in a circle.

Combinations: Order doesn't matter. You select the items without replacement (so the results can't repeat)

- You take the total number of ways to rearrange all n items (denoted n!), and divide by the number of ways you rearrange the n-k items you don't select [(n-k)!], you divide by the number of ways to rearrange the k items you do select (k!).

Suppose you have ten people in a class. You need to choose three students to win a prize, and all the prizes are the same (hence, the order in which you chose the three prizes isn't important). How many ways can you choose?

- You have 10!/(10-3)!=720 ways to rearrange and select three people from the class of ten.
- You don't care about the number of ways to arrange the three you select. So, you take 720 and divide it by 3! = 6 to get 120. You have 120 ways to choose three people from a class of ten for identical prizes, compared to 720 ways to choose three people where the prizes are different.

- There are k! more permutations than combinations when you choose k items from n items without replacement.

Probability problems involving combinations

Splitting objects or individuals into two groups

Suppose you have a group of ten friends going into a movie, and you need to split the individuals into two groups; one group will sit in the front row, and one group will sit in the back row. (Assume you don't care who sits by whom within a row.) How many ways can you split the groups?

- Because you don't care about who sits next to whom within a selected row, the order of the selected people doesn't matter. → COMBINATION
- You select 5 people from the ten without replacement when the order doesn't matter, so you find 10C5 = **252**

Suppose you want to pick five lottery numbers without any numbers repeating. You have a choice of 1 through 42 for each number. How many ways can you choose your numbers?

- Order doesn't matter, and you're selecting without replacement with no regards for order
- Because you want to select 5 numbers from the group of 42 without replacement with no regard for order, the total number of ways to choose is "42 choose 5" = **850,668**

Suppose you want to pick five lottery numbers and the numbers can repeat. You have a choice of 1 through 42 for each number. How many ways can you choose your numbers?

- Order doesn't matter, but you can repeat numbers
- "42 choose 5" (n+k-1)!/(k!(n-1)!)

Choosing k items from n distinct items

	Repetition is allowed	**Repetition is *not* allowed**
Order DOES matter	n^k	n!/(n-k)!
Order DOESN'T matter	(n+k-1)!/k!(n-1)!	n!/(k!(n-k)!)

Suppose you're playing 5-card poker, how many possible hands can you make?

- Order doesn't matter, repetition not allowed
- "52 choose 5" = **2,598,960**

How many ways can you pull out 5 cards from a deck and they both are hearts?

- Order doesn't matter, repetition not allowed
- "13 choose 5"

What is the probability of choosing a heart flush hand from a deck of cards?

- P= possible 5 card heart hands/possible 5 card hands